FACTORS PERTINENT TO CHILDREN'S RESPONSIVENESS IN SPATIAL PROBLEM-SOLVING ACTIVITIES

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THE NEED FOR RESEARCH INTO CHILDREN'S SPATIAL THINKING PROCESSES

Despite the research in psychology, on spatial abilities and its development (see Clements, 1981; Eliot, 1988; Lohman, Pellegrino, Alderton, and Regian, 1987), there is little evidence that ordinary school-based lessons, particularly in the primary school, can improve students' spatial thinking processes (Eliot, 1988; Lean, 1984). The term "spatial thinking processes" is used in this paper to include (a) visual imagery skills such as recognition of shapes, transforming shapes, and disembedding parts and shapes, (b) spatial conceptualising, and (c) the interaction of visual imagery with the early conceptualising of shapes.

If a study is to consider school-based learning experiences, then it is necessary to consider relevant factors of the classroom context. Several studies have shown the effectiveness of factors such as the use of concrete materials (for example, Suydam, 1986), and the use of language especially as it is involved in cooperative groups (for example, Dalton, 1985; Davidson, 1990). Yet there is some evidence that a dominant member of the group can prevent a correct solution being obtained (Stacey, 1989). On the other hand, advocates of teacher-led discussion emphasise the importance of the teacher challenging, focussing, and questioning (Cosgrove & Osborne, 1985). No matter how the experiences are provided, the issue of accessing and assessing spatial thinking processes needs considering and an overview of this issue was given in an earlier paper (Owens, 1990).

THE STUDY

The study reported in this paper consisted of two interrelated sections. In the first part of the study, the aim was to consider whether a series of spatial problem-solving activities would have a significant effect on performance on spatio-mathematical tasks. In addition, consideration was given to the effect on performance of other factors, namely classroom organisation, gender, and Year at school. An experimental design was used to investigate these effects.

In the second part of the study, consideration was given to the nature of the thinking processes, together with the effects of student-student and student-teacher interactions associated with different classroom organisations.

EXPERIMENTAL STUDY

Design

The study employed an experimental matched-group design in which subjects from the same class and in the same cluster as determined by their ranking on total pretest scores, were randomly allocated to one of three groups. Group 1 students worked individually on two-dimensional spatial problems and participated in whole-class, teacher-led discussions. Group 2 students worked in small cooperative groups on the same spatial problems but Group 3 students, who formed the "control" group, worked in small cooperative groups on a series of number problems. The same teacher (the author) taught all the students. Groups 2 and 3 were together in the classroom as they were both organised into cooperative groups. Group 1 students were taken separately but to maintain the same student/teacher ratio for all students, this group had twice as many students as the others.

All students completed a spatial thinking pretest, an introductory activity, and then ten spatial (or "control") activities over a six week period. After the learning sessions, the students completed a parallel posttest, and eight weeks later (six weeks for one school because of annual holidays) completed a parallel retention test. The testing consisted of items involving two-dimensional space and items requiring transfer of thinking to three-dimensional tasks. Analyses of covariance with pretest scores as covariates (PRE2D for the 2D tests but PRE2D and PRE3D for the 3D tests) were used to determine the effects of gender, Year at school (2 and 4), and Group (1, 2 and 3) on posttest and retention test scores.

Subjects

There were 190 students involved from three schools in Sydney with a high proportion of students with non-English speaking backgrounds. Both a Year 4 and a Year 2 class were involved from each school.

Test Instrument

Two parallel forms of a test were developed for the purpose of measuring spatial thinking processes. There are six subtests:

Subtest 1 requires respondents to recognise congruent shapes in rotated or reflected positions. Some of the items in this subtest are more easily solved by analytic than by holistic procedures.

Subtest 2 is concerned with recognition of shapes made by tessellations of a smaller shape.

Subtest 3 has three types of items. Two types have diagrams of configurations made from matches and respondents are asked to complete shape outlines by drawing the matches that need to be added, and, in the second type of item, to mark the matches that need to be taken away. For the other type of items respondents are required to recognise and trace over embedded shapes in outlines of configurations.

Subtest 4 is about the construction of models made of parts from memory and/or under transformation.

Subtest 5 contains items concerned with nets of three-dimensional shapes; in particular with the mental transformation of two-dimensional shapes by folding.

Subtest 6 is primarily concerned with the recognition of congruent angles in shapes in different orientations or contexts.

Following a Rasch analysis, there were 46 items used for the 2D test and six items used for the 3D test (subtest 5) (Owens, 1991).

The Learning Experiences

The spatial learning experiences involved ten sessions over a six week period (plus an introductory session) and covered a number of activities using commonly available materials. These materials were:

1. the seven-piece tangram - students were to find the similarities and differences of the shapes, make shapes from other shapes, make outlines of the shapes using sticks, sketch shapes and configurations, and compare and order the angles of the shapes;

2. pattern blocks - students were to make the shapes larger, compare angles of the shapes, and make the shape outlines with sticks;

3. pentomino shapes - students were to make them from square breadclips, find symmetries, and tessellate the shapes;

4. matchsticks - students were to make designs of squares and see shapes within designs.

The activities were designed to encourage the students to solve problems through discussion and the invoking of visual imagery and pertinent spatial concepts. The use of open-ended and multifaceted activities catered for the needs of students with a range of prior experiences and existing concepts. The activities were used to provide a basis for challenging students to reflect on and, whenever necessary, to modify existing concepts, images, and skills. In particular, it was expected that teacher/student and student/student interactions would encourage the use of language and communication to facilitate the further development of concepts.

Results

With the two spatial groups combined, analyses of covariance of the scores on the posttests (POST2D and POST3D) indicated no significant effect on the variance due to the three main variables or their interactions. However, as shown in Table 1, significant effects were shown on the two-dimensional retention test (RET2D). The improvement for the groups learning from spatial problems was significantly better on two-dimensional spatial thinking than the "control" group learning from number problems.

A similar effect was not found with the three-dimensional retention test scores (see Table 1). This is consistent with the study by Rowe (1982) who found that students in Grade 7 who undertook a series of lessons on either 2D or 3D tasks improved in their scores on 2D items but not 3D items.

Table 1: Analysis of Covariance of Retention of Spatial Thinking Processes; Spatial versus Number Groups

	two-dimensions		three-dimensions	
	F	signif. of F	F	signif. of F
main effects				
group	5.072	.026 *	1.389	.240
gender	0.037	.848	4.146	.043 *
year	0.080	.778	0.550	.459
2-way interactions				
group gender	0.647	.423	0.228	.634
group year	1.576	.211	0.719	.398
gender year	2.248	.136	0.408	.524
3-way interactions	0.011	.916	0.588	.444

Note: * significant at the 0.05 level

Table 1 also indicates that the amount of variance in scores on the 3D retention test explained by gender was significant at the 0.05 level. Males improved more than females except for the Year 4 girls who were learning as cooperative groups but none of the comparisons for groups by year were significant at the 0.05 level as the intragroup variance is larger than the between group variance. The gender and gender by classroom organisation effects need further investigation.

There were no differences in improvement between the two groups using the different classroom organisations (an analysis of covariance was used for the analysis comparing just the two groups involved in spatial activities).

THE QUALITATIVE STUDY

Purpose

Although the experimental study showed that spatial learning experiences did affect children's spatial thinking processes, it did not explain **how** the learning experiences helped students to develop their spatial thinking processes or **how** they solved the spatial problems. In order to determine the relevant factors and their relationships, the qualitative study was undertaken. Earlier training studies did not provide this kind of data and did not explain how the children's spatial concepts, imagery, and other cognitive processes were used and developed.

Categories pertinent to spatial problem solving were developed from written records of tertiary students and observation of primary students working individually (Owens, 1990). These categories were the basis of the analysis using data from twelve children (four groups of three) working individually near others or as a cooperative group on all the problems and children in classrooms in Australia and Papua New Guinea.

Procedures Used in the Qualitative Study

In order to carry out this study, it was necessary to use mainly observational data, with the use of video-recordings, to decide on apparent cognitive processes. Some verbal support was given to these observations either by the comments students made while problem solving or in recalling their thinking on viewing the video playback. Each incident in which a significant development occurred was described and then categorised. The categories and subcategories used for aspects of interaction and cognitive processing are given in Appendix 1. The names of the major categories of cognitive processing, that is, imagery, concepts, heuristics, and affective processes, were influenced by Goldin's (1987) model of problem solving. The names of the visual imagery subcategories - concrete, dynamic, action, pattern, and procedural - were mostly influenced by Presmeg (1986) but modified to accommodate the approaches suggested by information processing theories, the comments of students, and the observational data. Other researchers, with experience in this field, agreed with the kind of cognitive processes indicated by certain behaviours and certain incidences as recorded on the video-tapes. The exploratory nature of this study into factors which influence spatial learning required a holistic approach and episodes were considered as a whole in deciding the best way to describe the learning.

The Importance of Interactions and Responsiveness

Based on the analysis of the video-data, the following description of students' participation in spatial tasks was developed. When a student was engaged in spatial problem-solving activities, there were inevitable interactions between the person, the influences of the context on the person and, in turn, the person's responsiveness. This is illustrated in Figure 1.

The cyclical interactions continued and changed throughout the period of engagement in the activity. Interaction normally encouraged further responsiveness as the learner became increasingly more involved in the problem-solving activities. There was a "snowballing" effect, not only on participation, but also on the extent and quality of imagery, concepts, understandings, and tactical approaches. Students would frequently monitor their own progress towards a solution.

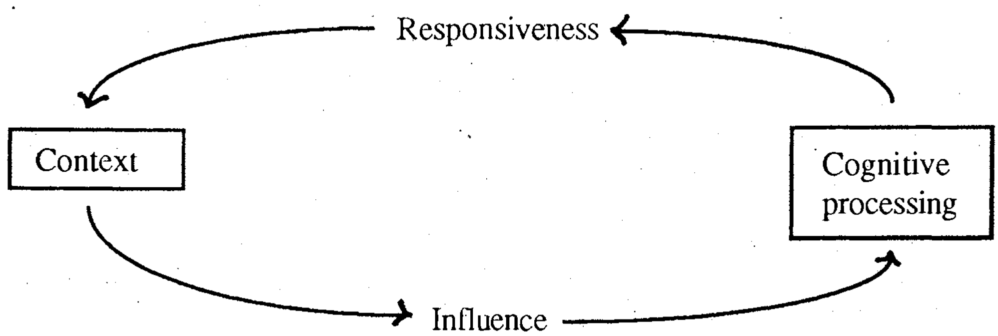


Figure 1: Interaction between context and cognitive processing

The term "responsiveness" is used to suggest that students responded to the task, the materials, and the other components of the environment by becoming actively involved in the problem-solving activity. Responsiveness implies a degree of understanding of the situation as well as involvement and interest in the activity. Responsiveness is associated with a two-way relationship, that is a dynamic interplay between a student and the environment which comprises fellow students, the teacher, the classroom, the task, and the materials. For the student, there is a cycle of thinking, feeling and responding, then further thinking, feeling and responding.

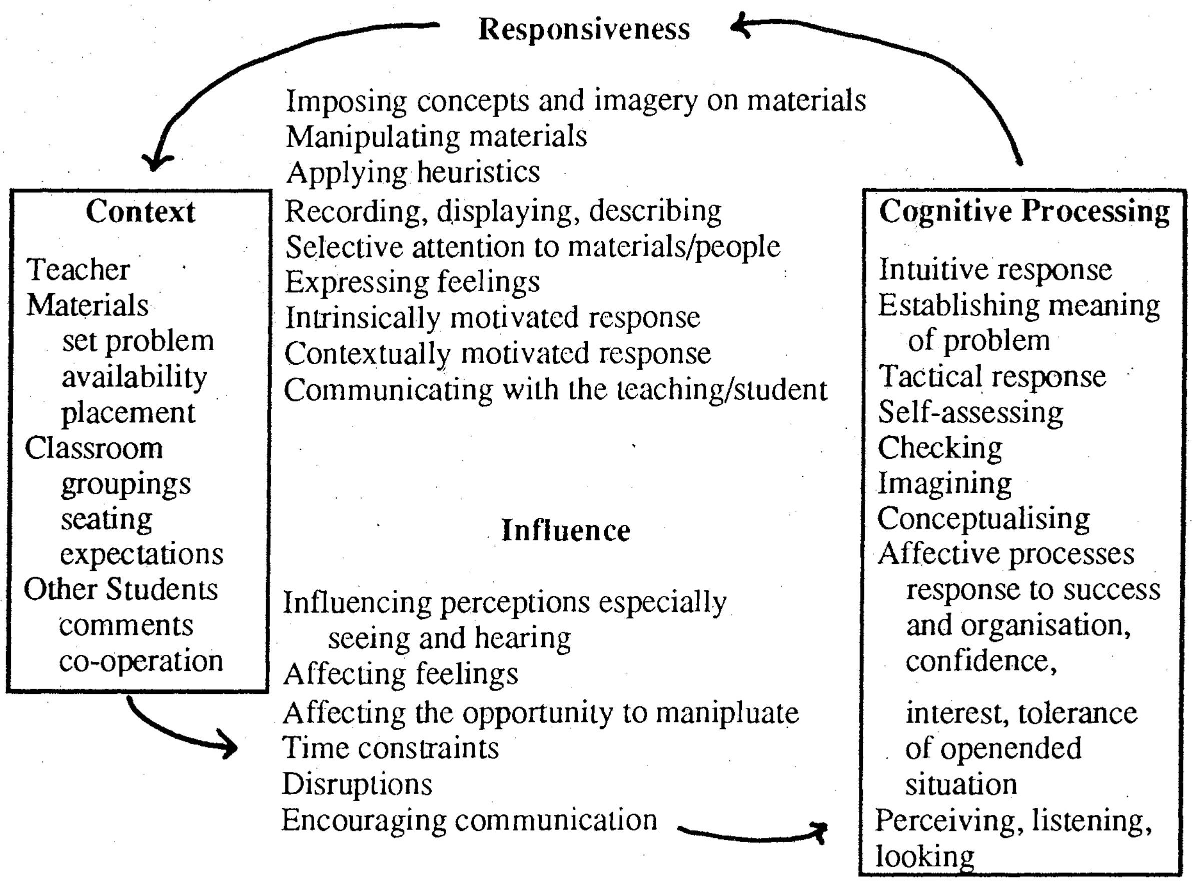


Figure 2: Aspects of the factors involved in the problem-solving process.

The typical response by the children to a task was one of engagement with the materials. Although there was some degree of uncertainty about what was expected, students were positively motivated to participate, engagement was maintained because of interaction with the materials and with others. This was the case despite differences in confidence, intuition, spatial thinking, and classroom organisation.

The responsiveness of students to the overall environment was a core phenomenon to which other phenomena can be related. Figure 2 lists aspects of each of the concepts used in Figure 1.

Several excerpts from the descriptions of the video-data of some of the Year 2 children will be used to illustrate particular aspects of responsiveness, namely: (a) how children imposed their concepts and images onto the materials that they were manipulating, (b) how they established meaning and tactics for the activity, (c) how they monitored their progress, and

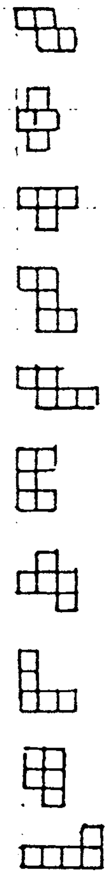
(d) how they expressed their feelings. Each episode captured on video is described in the present tense as it happened. Numbers are used to simplify reference to particular incidents.

Responsiveness - Its Causes and Effects

Each task in the investigation involved the use of materials and, therefore, actions with these materials assisted students to construct concepts and visualisations.

Although James was part of a cooperative group, he began the second spatial activity somewhat competitively. He was thoroughly involved in making new shapes from four square breadclips and then in making pentominoes from five squares. He also enjoyed commenting and in other ways expressing his achievements and feelings of pleasure.

1.01 James continues to count how many he has made, comparing his number with his friend's number.



1.02 Using four squares, he makes a Z, checks that it is all right and then makes a cross avoiding repeating the Z

1.03 His friend points out "it is half here," so he changes it to a T.

1.04 He begins with five squares, deliberately positioning the pieces to make a Z. Then he makes a lineZ.

1.05 He notes his friend's shape saying "yours has three columns, mine has two; she copied me". (The same shapes were made in different orientations.)

1.06 Despite the teacher suggesting that they work together, he keeps making shapes quickly and happily, commenting on how well he is going. He uses a tactic of beginning a new shape with three-in-a-line. He counts his shapes and says "I'm beating her." He knows what he is making before he completes the shape, showing joy before he finishes making the shape. He places three-in-a-row, and claps as he makes a C.

1.07 He cannot recognise the "odd" shape in different orientations despite moving his body to assist orientation. Otherwise he changes the shapes to make easily-recognised shapes such as L3, and the near-square comparing the incomplete shapes with his short-term memory images of those he has made (that is, he is not physically glancing at his shapes).

1.08 He changes his tactic from starting with three-in-a-line to beginning with four-in-a-line.

1.09 He quickly grabs the last five breadclips so that he can make another shape.

1.10 He wants to make a car but ends up with lineZ, considers it, and says "Oh, I can't make any more." His degree of activity wanes as the teacher points out the repeats in the group's work.

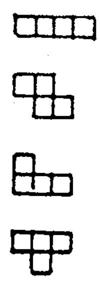
1.11 He recognises the repeated lineZ and L4.

There are several points to note about James' responsiveness. First, a friendly competition existed between the students and this motivated him to participate and achieve (para. 1.01 and 1.06). Certain affective characteristics are evident in his behaviour - his responses to his successes (para. 1.01 and 1.06), his competitiveness (1.01, 1.06, and 1.09), and his loss of interest at the end (para. 1.10). Second, James' use of imagery influenced his responsiveness - not only his manipulation of materials (para. 1.03, 1.04, 1.07 and 1.10) but also his comment to his friend (para. 1.05) and his self-assessments (para. 1.06, 1.10, and 1.11). His imagery helped him to stay on task (para. 1.06 and 1.10). Third, he assessed or monitored his own progress on the task and this, too, influenced his responsiveness. He expressed how he was progressing (para. 1.01 and 1.06), and he changed his tactic in an appropriate way (para. 1.08 and 1.10). Finally, he expressed his understanding and knowledge (para. 1.03, 1.04, 1.05, and 1.11). The changes in his cognitive understanding and his responses (para. 1.03 and 1.10/11) were precipitated by comments to him by his friend and the teacher. Thus we note how his understanding of the problem, his visual imaging together with student and teacher comments, his self-assessment, and his attitudes affected his responsiveness. At the same time, we can note how his visual imaging and tactics improved. In contrast, the imagery used by another student, Sam, was global while he was working on making outlines of the pattern-block and tangram shapes, although he showed some skill in getting sides with the correct slope. He chose simple shapes to start with, and juggled the trapezium and parallelogram shapes as if he was dependent on a global image.

Responsiveness Varies during Problem-Solving Activities

Responsiveness varies throughout the problem-solving activity as illustrated by the following excerpt about Lois. Her initial responsiveness was typical of students beginning a problem; that of unease. But she continued actively, making shapes and changing them until she seemed to work out what she was required to do to solve the problem, and then she systematically continued to make shapes. Lois was asked to make as many different tetromino shapes as possible with four square breadclips. She had her own pile of breadclips and she was working next to her friends.

3.01 Lois makes a line turning pieces; she is uneasy. She looks at her friend who has made a line, then an L but he changes it. She hears her friend say "I've made a pattern. I know, I'll make a square." She makes another line and looks at her friend's work.



3.02 Then she makes two Z's in different orientations and gives her characteristic "erm"- a quiet exclamation of joy.

3.03 As usual, she begins a new shape with two squares and says, "I know." She temporarily makes an L but changes it to a T.

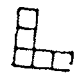
3.04 She listens to the teacher ask her friend if he has some shapes that are the same, and she leans over to point out her friend's two T's. When he points out her two Z's, she wants to leave them; "that's the up way", she claims and when he mucks one up, she is cross.

3.05 She makes several L's calling one "a flag" before she has made it.

3.06 She listens to the teacher discussing with her friend why he didn't make the square but she doesn't make a square herself.

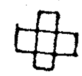
3.07 Then the teacher tells them to begin again making shapes with five square breadclips at a time. She quickly falls into the same tactic that she previously used of putting down two pieces, adding a third and then with a piece in each hand systematically moving the pieces around until she sees a new shape. The first shape she makes is a T upside-down.

3.08 She puts down two squares, and thinks about where to place the third leaving it so she has the three-in-a-row. She then places a fourth to make an L and then temporarily places the fifth to form the square-like shape before shifting it to make L3.

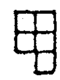


3.09 She looks at her friend's shapes.

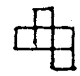
3.10 Next she uses two again, the third in a three-in-an-L and with a piece in each hand, places them to make the square-like shape. However, she is not happy with this and proceeds to place them to form a cross. She gives another expression of joy, "erm".



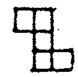
3.11 She quickly collects some more breadclips. She places, two, the third in an L, the fourth to make a square and the fifth to make the square-like shape. This time she leaves it.



3.12 She collects more breadclips; she places two as usual, then the third to make the L, and then two together to make the odd shape, which she decides to keep.



3.13 She continues on as if settled into a routine, occasionally looking at her friend's shapes. She makes the square-like one upside down, contemplates it and moves the last piece to give her a Z which she checks is not the same as the odd shape.



Lois' responsiveness was a little guarded at first (para. 3.01) and she looked at her friend's work for reassurance. Even after discussions with her friends, she appeared to have her own agenda judging by what she kept and what she discarded as inappropriate (para. 3.03, 3.04, and 3.06). However, she continued with a fairly consistent tactic making shapes and checking their acceptance (para. 3.07 and on). There was slight competition between her and one of her friends. Although she worked independently, she appeared comfortable with looking at her friends' work and making comments to them (para. 3.04). Imagery appeared to play a part in her work guiding her manipulation of pieces and her creation of shapes that she named or described (par. 3.03, 3.04, and 3.05). Imagery involved the evocation of holistic pictures, but the imagery became more operational, that is consisted of movements of pieces, and more analytical as the problem-solving continued and she evaluated whether a shape was acceptable. She appeared to be working to images of shapes although she sometimes rejected these shapes after making them (para. 3.03 and 3.05).

Some General Comments

Among the cognitive proceses that developed during the problem solving were visual imagery, conceptual processing and heuristic processing. All the students were keen to play with the materials to make shapes; they tried to meet the problem as set by the teacher; and they interacted with the teacher or other students if they were unsure of what to do or if they were enjoying their success. It was clear that students were manipulating materials deliberately; that is there was some thought directing the actions. As they worked, their manipulations suggested that they were becoming more systematic and more analytical as their imagery and ideas developed. They often monitored their own thinking and progress.

The two different organisations of students did not determine a clear pattern of student-student and student-teacher interaction for there was considerable variation in the amount of interactions with the teacher and fellow students. However, the spreadsheet analyses did show some differences between the groups. For example, there were more comments to the teacher based on holistic imagery and concepts in the classrooms where students were working individually. In addition, student-initiated comments to fellow students and the teacher were high in both groups and were generally either expressing affective factors or thoughts that were more analytical than holistic. There were more comments in the cooperative groups on what to do with pieces and who should be using the pieces.

CONCLUSION

The study indicated that children use a wide range of thinking processes in solving spatial problems and that classroom spatial activities improve and enrich students' spatial thinking processes. Among the thinking processes involved in problem solving are visual imagery, conceptualising, and heuristics. Their concepts, both relating to appropriate actions and to shapes, and visual imagery influenced their attention to details of the materials. Students evaluated their progress through the problem and frequently expressed their self-assessment. Affective processes, in particular enjoyment at success and attitudes to the group, also played a part in their responsiveness. Of particular importance was the effect of interactions with both the materials and other people in encouraging students' responsiveness and their development of spatial and other thinking processes.

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